Redesign of a Tennis Racket for Reduced Vibrations and Improved Control and Power

MENG 370 Design Project Report
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Abstract

Within the constraints of modeling with a cantilever beam and not focusing on several variables, we examined the effects of instituting an internal damping system in a mid-grade tennis racket. Using a simple testing method (one that involved LabView programming, accelerometers, and a string vibration dampener) allowed the experimental data to easily be obtained. While there were some sources of error, the parameters that were focused on made for minor losses. A key area that was investigated was the vibration/mathematical model, which was extracted from the experimental results and a basic cantilever deflection system. The mathematical model ultimately showed how an undamped system would be phased by damping and the relation through the equations therein.
Introduction

The objective of this design project is to explore the effects of internal damping systems within a tennis racket. Tennis rackets currently use external damping systems that consist of any kind of soft material, especially elastomers, tied to the strings. These external dampening systems primarily focus on reducing vibrations in the string bed. Internal damping systems would focus on reducing vibrations within the frame of the racket directly.

Vibration dampening systems are widely used in the world of professional tennis. Dampeners reduce the shock generated when the tennis ball hits the tennis racket string bed, which, in turn, reduces the vibrations that go through the player’s arm. Vibration dampeners can decrease the risk of elbow injuries caused by racket vibrations. In this project, an innovative internal dampening system is designed.

Another motivation for improving on dampening systems of a tennis racket is to allow professional players to increase weight and stiffness for increased performance while maintaining a low level of vibrations. This is a problem in racket design because increasing stiffness transmits more of the shock load to the arm, while heavier rackets vibrate less. Heavier rackets and stiffer rackets both generate more power and have a larger sweet spot. The goal of this project is to effectively create a dampening system that will absorb nearly all of the vibrations in the racket, while improving performance during a tennis match.

A racket designed for professional use is considered. Such rackets are stiffer and heavier than entry-level rackets. To get a better understanding of the weight concept, an explanation is provided. Entry-level rackets have weights in the range of 255-275 grams, while professional tennis rackets weigh between 290-350 grams. When the racket is heavier, the motion is easier to execute due to the added moment of inertia, but it requires a lot more strength in the arm for this motion to be fully controlled. A beginner who uses a heavy racket is most likely to develop elbow or shoulder injuries due to the uncontrolled swings.
In tennis rackets, carbon fiber composites are typically used in the frames of rackets. These composites may include materials such as graphite, Kevlar, and aramid. Using materials like these make the racket a lot lighter and allows it to be strung at an extremely high tension. Composite materials are materials that are composed of multiple material types, done through intense heating and pressure in order to change the chemical structure of the materials. Composites are extremely light weight, elastic, have a high strength and have high corrosion strength. In comparison to steel, a typical composite material would be about a quarter of the weight, but nearly four times stronger. However, composites are extremely expensive and in most instances are irreparable after fracture.

In this project, however, a single material will be assumed for the frame, with all characteristic data presented accordingly. The use of a uniform, isotropic, and homogeneous material will also simplify the problem at hand as opposed to a composite material made up of multiple materials with varying compositions. The racket system itself is viewed as a single plane where the only vibrations analyzed are those perpendicular to the head of the racket and in the longitudinal direction. This becomes more evident during testing, where the racket is clamped and a ball is dropped vertically onto the racket.
Data and Methods

Testing

Introduction

An important part of our project is the testing of real tennis equipment. We have the need to compare our theoretical mathematical model to real practical data. In order to do so, we have designed a very simple, yet useful, experiment that gives us some important information about the vibration of an actual tennis racket. The experiment aims to calculate the vibration acceleration at different spots on the racket using simple accelerometers. It is also a great opportunity to test for the efficiency of regular string dampeners. This testing procedure makes use of LabView for the receipt of data from the accelerometer.

Objectives

The experiment has only a couple of objectives, yet very important in our design process:

- Record and display the acceleration of the frame due to impact vibrations at a few spots around the racket.
- Measure the height at which the ball bounces in order to calculate the impact energy for the mathematical model.

On the other hand, we are not testing for:

- The displacement of the frame due to vibrations.
- The strings vibrations.
- The vibrations of the frame with our designed dampening system because of limited budget and material.

Equipment

Very little equipment is necessary for this experiment:

- A tennis racket. A “Head MicroGel Radical Midplus” model racket is used in this case. This racket is an average high-level racket. The characteristics of this racket, listed below, are also used when computing the mathematical model and when creating the 3D design on Catia V5:
  - Length: 69 cm
- Strung mass: 312 g
- Cross section: 2 cm x 3 cm at the base of the throat
- Any additional information can easily be found on tennis equipment retailer websites

**A string dampener.** We use the simplest form of dampener possible that a lot of tennis players use, which is a simple regular rubber band tied around the 2 central vertical strings at the bottom of the string bed as shown in Figure 2.

- A regular tennis ball.
- A 1 meter long yardstick.
- An accelerometer.
- A computer equipped with LabView
- Two fixed clamps.

**Procedure**

The testing procedure itself goes as follows:

1. It begins with the programming of LabView. The LabView program is set to record the acceleration, as measured by the accelerometer, and plot it versus time.

2. After programming LabView and having it ready, we can set up our experiment. First, we fix the racket horizontally, with the plane being horizontal too, using the fixed clamp. The racket is set with the clamp positioned at the extreme bottom end of the grip to simulate the player’s hold of the grip as realistically as possible. Then, using the other fixed clamp, we set up the yardstick vertically with the zero point at the height of the frame. See Figure 3 for the actual setup.

3. Using the wax provided, the accelerometer is stuck at Point 1 as shown in Figure 2. On LabView, we start recording. We drop the ball from a height of 1 meter above the center of the string bed, or what is commonly called the “sweet spot”, as shown on Figure 4. The ball rebound height of the ball is recorded, and the ball is caught before it bounces a second time. This is repeated three additional times before stopping the record on LabView.

4. We repeat step 3 for each point shown in Figure 2.

5. We can then set the string vibration dampener on the strings at the indicated position and repeat steps 3 and 4.
Possible sources of error

Although very simple, the experiment involves a lot of sources of error, uncertainty, and lack of precision:

- Absorption of vibrations and energy through the grip fixation. Although we try to close the clamp on the grip as tight as possible, there might be some losses due to the soft material the grip is made of.
- Absorption of vibrations through the wax.
- Approximate height of drop and bounce.
- Approximate spot of bounce.
- Presence of strings. For ease of calculation, our mathematical model omits the strings. Instead of having an impact on the strings, we consider our impact to be directly on the racket.

Results

Our graphical results can be seen in Appendix B.

The height of rebound was consistent through all iterations, with and without string dampener. The height was found to be about 15 cm.
**Mathematical Model**

**Objective**
The objective of the mathematical model is to allow us to demonstrate the effects of added damping systems within the tennis racket. Ideally, we would have multiple rackets, apply different damping techniques to each, and test according to the testing procedure outlined above. However, due to limited resources, a simplified mathematical model should show the effects of the damping.

**Assumptions**
In order to simplify the model, however, a few assumptions must be made. The primary assumption is that the strings of the racket are to be ignored. Due to the fact that there are vibrations within the strings, the system becomes too complex. Instead, we take the racket to be a cantilever beam with a fixed end. Moreover, the racket is assumed to be solid with a uniform cross-sectional area along the entire length. The impact of a tennis ball on the racket is assumed to be a force (P) applied at the ‘sweet spot’. The notion of the cantilever beam that is fixed at one end acting as the tennis racket allows for several variables to be investigated, and in-turn, applied to the problem. These variables include: Young’s Modulus of Elasticity (E); the length (L) of the beam; the moment of inertia of the beam (I); and the selected material density (ρ). It is then appropriate to assume that a deflection and vibration analysis can be deduced from the aforementioned ideas.

The density (ρ) and Young’s Modulus of Elasticity (E) of the material were estimate values given by Davis (2010). The force (P) applied at the ‘sweet spot’ was calculated using the velocity equations and the rebound height of the tennis ball. We also assume only the first node of vibration to be relevant in our calculations, as this was the vibration seen in LabView during our experimentation.

**Calculation**
Whereas many assumptions are made to model the damping effects of the racket, the formulas that lie therein bring the vibration aspect of the model to light. The simplifications that are made yield a fourth degree differential equation that allow for a complete analysis of the cantilever beam which can be seen in Appendix C.

Once the vibration and acceleration models are found, the amplitude can be found. This amplitude is then used in the damped model. We adjust the damping coefficient (ζ) to fit the mathematical model to the experimental data. The graphical results for each point can be found in Appendix D.
Discussion

The mathematical model we found is a very simplified version of what is really happening within the system. We assume a very simplified structure and shape, we assume homogeneous, isotropic, and uniform material and material properties, and we assume no vibration motion within the strings. With these assumptions, we are able to get a model, but it is very subjective. We adjust the coefficients within the model to match our experimental data. Using this method, we are able to match our model to our experimental data. This is done with the assumption that we can now adjust our model to predict the effects of added dampening systems.

In the model, the original damping coefficient is .04. By increasing this number, the vibrations dampen out even quicker. Different vibration dampening materials have been researched. Information on the dynamic flexural behavior of some materials at 0.2 Hz has been found.

According to the table presented in the article “Materials for vibration damping”, by D. D. L. Chung (more information on the references section), the material that has the greatest capability to minimize vibrations is Al-AIN. Al-AIN is a material composed of aluminum powder and nitriding gas. This material appears to be very expensive. The manufacture cost of rackets would drastically increase, and so would their retail price. Therefore, this material cannot be used in a tennis racket.

The next material with the higher vibration dampening qualities is a carbon-fiber epoxy matrix composite (without interlayer). This material is easily applicable to tennis rackets, because these are already made of carbon-fiber. The manufacturing process would remain the same, and only some epoxy would have to be added to the carbon-fiber composite. Epoxy is not an expensive material, so the production cost will remain virtually the same.

Due to technical reasons, it is not possible for this project to build a tennis racket with the carbon-fiber epoxy matrix composite, so further testing cannot be completed. For practical purposes, it can be concluded that the addition of epoxy to the composite will severely reduce the vibrations of the racket.

Our model was helpful, however, in observing the vibrations and accelerations at different points along the racket. Using the accelerometers at different points, we were able to see how the original amplitude dropped as we moved close to the gripped end of the racket.
The other thing noticed through our experiment was the effects of the string dampener. As we expected, most of the vibration was within the strings. When the external string damper was introduced, the vibrations dropped off to nearly zero in less than half of the time it took the undamped system. This was the expected result.
Conclusions

Unfortunately, our testing and modeling did not yield the results we were hoping for. We were able to match the model to the experimental results, and we are able to see that increasing the damping coefficient greatly reduces the vibrations. However, our model does not allow us to quantify the amount of vibration reduction for different materials. The best way to determine the reduction in vibrations due to an internal damping system is to actually implement the internal system and run tests. However, we were not able to purchase multiple rackets, to run these tests on.

Potential additional areas of research could include bettering the mathematical model by reducing the number of assumptions and simplifications used.
Appendices

Appendix A – Testing Setup

Figure 1 - Front, Right-Side, and Top Views

Figure 2 - Flat View of our Model Racket with the Different Testing Spots and the Location of the String Dampener
Figure 3 - Testing Setup
Figure 4 - Testing and Recording
Appendix B – Testing Results

Figure 5 - Point 1

Figure 6 - Point 1 Dampened

Figure 7 - Point 2
Figure 11 - Point 4

Figure 12 - Point 4 Dampened
Appendix C – Mathcad Worksheet for the Vibration Model

\[ P = 3.66 \, N \]
\[ \rho = 2000 \, \frac{kg}{m^3} \]
\[ A = b \cdot h = (6 \cdot 10^{-4}) \, m^2 \]
\[ L = 0.69 \, m \]
\[ S = 0.5 \, m \] 'sweet spot' location
\[ E = 90 \cdot 10^9 \, Pa \]
\[ I = \frac{b \cdot h^3}{12} = (2 \cdot 10^{-8}) \, m^4 \]
\[ \lambda = \rho \cdot A \]
\[ kL1 = \begin{bmatrix} 1.875104069 \\ 4.694091133 \\ 7.854757438 \\ 10.99554073 \\ 14.13716839 \\ 17.27875953 \end{bmatrix} \]
\[ kL = kL1_0 \]
\[ k1 = \frac{kL}{L} = 2.718 \, \frac{1}{m} \]

natural frequency
\[ \omega = 2 \sqrt{\frac{(k1)^4 \cdot E \cdot I}{\lambda}} = 286.021 \, \frac{1}{s} \]

Figure 13 - Constants used in the Mathematical Model
natural frequency
\[ \omega = \sqrt{\frac{(k_1)^4 \cdot E \cdot I}{\lambda}} = 286.021 \]

\[ C_2 = 0.5 \]

\[ X(x) = C_2 \left( \cos(k_1 x) - \cosh(k_1 x) \right) + \frac{-\cos(kL) - \cosh(kL)}{\sin(kL) - \sinh(kL)} \left( \sin(k_1 x) - \sinh(k_1 x) \right) \]

\[ v_0(x) = \begin{cases} 
-\frac{P \cdot S}{E \cdot I} \left( \frac{x^2}{2} - \frac{x^3}{6} \right) & \text{if } x \leq S \\
\tan \left( \frac{-P \cdot S}{E \cdot I} \left( S - \frac{S^2}{2} \right) \right) \cdot x + \frac{-P \cdot S}{E \cdot I} \left( \frac{S^2}{2} - \frac{S^3}{6} \right) \cdot S - \tan \left( \frac{-P \cdot S}{E \cdot I} \left( S - \frac{S^2}{2} \right) \right) & \text{else}
\end{cases} \]

\[ A_2 = \frac{2}{L} \int_0^L v_0(x) \cdot X(x) \, dx = 1.815 \cdot 10^{-4} \]

\[ v_0(L) = -1.33 \cdot 10^{-4} \quad X(L) = -3.337 \]

Figure 14 – Calculation of the Mathematical Model - Part 1
Figure 15 – Calculation of the Mathematical Model - Part 2
\[ \zeta := 0.04 \quad \phi := 0 \quad C := 2.75 \cdot 10^{-4} \]

\[ v_d(t) := C \cdot e^{-\zeta \cdot \omega \cdot t} \cdot \sin(\omega \cdot t + \phi) \]

\[ a_d(t) := \frac{d^2}{dt^2} v_d(t) \]

Figure 16 - Calculation of the Mathematical Model - Part 3
Appendix D – Graphs of the Mathematical Model

\[
\begin{align*}
\zeta &= 0.04 \\
\phi &= 0 \\
C &= 6 \times 10^{-4} \\
v_d(t) &= C \cdot e^{-\zeta \cdot \omega \cdot t} \cdot \sin(\omega \cdot t + \phi) \\
ad(t) &= \frac{d^2}{dt^2} v_d(t)
\end{align*}
\]

Figure 17 - Plot of the Acceleration at Point 1
\[ \begin{align*}
\zeta & = 0.04 \quad \phi = 0 \quad C = 2.75 \cdot 10^{-4} \\
v_d(t) & = C \cdot e^{-\zeta \cdot \omega \cdot t} \cdot \sin(\omega \cdot t + \phi) \\
ad(t) & = \frac{d^2}{dt^2} v_d(t)
\end{align*} \]

Figure 18 - Plot of the Acceleration at Point 2
\[ \zeta = 0.04 \quad \phi = 0 \quad C = 1.2 \cdot 10^{-4} \]

\[ v_d(t) = C \cdot e^{-\zeta \cdot \omega \cdot t} \cdot \sin(\omega \cdot t + \phi) \]

\[ a_d(t) = \frac{d^2}{dt^2} v_d(t) \]

Figure 19 - Plot of the Acceleration at Point 3
Figure 20 - Plot of the acceleration at Point 4
Appendix E – References


Haran, Second-Order Systems: Vibrating Cantilever Beams [PDF document].


